**Virus:** A virus is a small infectious agent that replicates only inside the living cells of an organism. It itself is not capable of reproduction, but if put into the right environment it can manipulate a cell to generate numerous copies of itself. The cell may devote all its resources to produce new virus particles and die after everything has been turned into virus.

But viruses can also have more subtle programs then this. They may tell the host cell to produce them, but only at a slow rate not endangering the survival of the cell. They may enter to a cell, insert their genetic material into its genome and be very quiet for a long time. Under specific conditions they may become reactivated and demand their reproduction. Other viruses once inserted into the genome of the cell may induce the cell to divide thereby producing two infected daughter cells. Such viruses may derive their host cell into uncontrolled multiplication and thereby cause cancer.

**Life cycle of viruses:** The life cycle of viruses can be subdivided into eight key events:

1. The virus attaches itself to the host cell.
2. The virus invades the cell.
3. The genetic material of the virus is uncoated.
4. Viral proteins are produced which start manipulating the host cell.
5. The viral genome is multiplied by the machinery of the host cell.
6. Further viral proteins are produced which will be used for the coat and possibly envelope of the new virus particles.
7. The new virus particles are assembled.
8. The new virus particles are released from the host cell.

**Immunity:** In biology, immunity is the balanced state of multicellular organisms having adequate biological defenses to fight infection, disease, or other unwanted biological invasion while having adequate tolerance to avoid allergy and autoimmune diseases.

**Cells:** The cell is the basic structural, functional and biological unit of all known organisms. A cell is the smallest unit of life. Cells are often called the “building blocks of life”. The study of cells is called cell biology or cellular biology.

**Basic Reproduction Ratio/Number:** The basic reproduction number  is used to measure the transmission potential of a disease. It tells us the average number of people who will catch a disease from one contagious person. It specifically applies to a population of people who were previously free of infection and haven’t been vaccinated. If a disease has an R0 of 18, a person who has the disease will transmit it to an average of 18 other people, as long as no one has been vaccinated against it or is already immune to it in their community.

The following factors are taken into account to calculate the of a disease:

### 1). Infectious period: Some diseases are contagious for longer periods than others. The longer the infectious period of a disease, the more an infected person is to spread the disease to other people. A long period of infectiousness will contribute to a higher value.

### 2). Contact rate: If a person who is infected with a contagious disease comes into contact with many people who aren’t infected or vaccinated, the disease will spread more quickly. If that person remains at home, in a hospital, or otherwise quarantined while they’re contagious, the disease will spread more slowly. A high contact rate will contribute to a higher value.

If an infected person makes  contacts per unit time producing new infected persons with a mean infectious period of , then the basic reproduction is defined as,

.

Three possibilities exist for the potential spread or decline of a disease, depending on its **** value:

1. If ****, each existing infection causes less than one new infection. In this case, the disease will decline and eventually die out.
2. If ****, each existing infection causes one new infection. The disease will stay alive and stable, but there won’t be an outbreak or an epidemic.
3. If ****, each existing infection causes more than one new infection. The disease will spread between people, and there may be an outbreak or epidemic.

**Basic model of virus dynamics/ a simple non-linear dynamics:** The basic model has three variables such as susceptible uninfected cells, infected cells and free viruses. In this simplest model, the basic principles are as follows (Figure-a). Susceptible uninfected cells are infected when they meet free viruses. Infected cells produce new virus particles that leave the cell and ﬁnd other susceptible target cells. Repeated rounds of infection result in the growth of the virus population. Growth is limited by the availability of target cells.

Figure-a: from sheet

If the population sizes of susceptible uninfected cells, infected cells and free viruses are ,  and  respectively, then the model may be defined as

 

 

 

Uninfected cells are produced at a constant rate  and die at a rate . When these susceptible cells encounter with free virus particles, they become infected at a rate , where the constant  describes the efficiency of the process. The infected cells die at a rate , which is the viral caused cell death. Infected cells produce new virus particles with a rate and the free virus particles that have been released from the cells decay with a rate . Thus, the average lifetime of an infected cell is ; the average lifetime of a free virus particle is ; the total number of virus particles produced from one infected cell (the burst size) is .

In addition to the dynamics describing virus infection, we have to specify the dynamic of the uninfected cell population. The simplest assumption is that uninfected cells are produced at a constant rate  and die at a rate . The average life time of an uninfected cell is therefore . In the absence of an infection, the population dynamics of host cells is given by



This is a simple linear differential equation. Without viruses the abundance of uninfected cells converges to the equilibrium value .

**Epidemic models:** Let there is a disease in a given population at time . Then the population can be divided into three classes:

1. , the number of susceptibles who may be infected by the disease.
2. , the number of infectives who has the disease and can transmit it.
3. , the number of those removed from the population by recovery, immunization, death, hospitalization or by any other means.

There are some epidemic model such as:

1. **SI model:** This is a simple deterministic model without removal. It has only susceptible and infected classes. Since, the number of susceptibles decreases and the number of infected increases due to infection so the rate of decrease of susceptiblesand the rate of increase of infected both are proportional to the product of susceptibles and infected persons. The model is defined as,



whereis a constant parameter. It is called infection rate.

1. **SIS model:**This is another simple deterministic model without removal. It hasalso susceptible and infected classes. In this model, a susceptible person can become infected at a rate proportional to and an infected person can recover and become susceptive again sat a rate . The model is defined as,



whereand are a constant parameters.

1. **SIR model:**This is a simple deterministic model with removal. It has susceptibles, infected persons and removal. In this model, the susceptibles can become infected at a rate proportional to and the infected persons can be removed at a rate . The model is defined as,



whereis the infection rate and is the removal rate of infectives.

**Question-01:** Discuss the SI epidemic model and show that the persons are ultimately infected.

**OR**

Discuss the deterministic models without removal and show that the infection spread throughout the population.

**OR**

In the epidemic model ,  in a closed population without removal, show that the persons are ultimately infected and that the rate of appearance of new cases rises rapidly to a maximum at time  and then falls to zero.

**Answer:**Let  be the total population size,  be the number of susceptibles and  be the number of infected persons in the population.

Since there is no removal in the population so we have



Let be the initial number of susceptibles in the population in which one infected person has been introduced so that



Due to infection, the number of susceptibles decreases and the number of infected persons increases. We assume that the rate of decreases of susceptibles or the rate of increases of infected person is proportional to the product of the number of susceptibles and the number of infected. So the epidemic model is





where is proportional constant which is also called infection rate.

From (3), we have











Integrating,









By using the initial conditions (2) in (5), we have







Putting the value of  in (5), we get



Again from (4), we have











Integrating,











By using the initial conditions (2) in (7), we have







Putting the value of  in (7), we get



From (6) & (8), we have



and

Thus, ultimately all persons will be infected.

By putting the values of and  from (6) and (8) in (3), we get





The equation (11) is the equation of a curve which gives a relation between and . This curve is known as epidemic curve. The above curve is a symmetrical unimodal curve with maximum at  where  is obtained from (11) by



















Thus, maximum time .

Thus, the rate of appearance of new cases rises rapidly to maximum at a time, depending on and , and then falls to zero.

**Question-02:** Discuss the simple SIR epidemic model. Find the condition on which the infection (disease) will ultimately die out or spread throughout the population.

**Answer:**Let  be the total population size,  be the number of susceptibles, be the number of infected persons in the population and  the number of those removed from the population by recovery, death or by any other means. The progress of individuals is schematically represented by



Due to infection, the number of susceptibles decreases and the number of infected persons increases. Thus for preparing mathematical model the following assumptions are to be considered.

1. The gain in the infective class is at a rate proportional to the number of infective and susceptibles i.e.  where  is a constant parameter.
2. The rate of removal of infectives to the removed class is proportional to the number of infectives, i.e. , where c is a constant.
3. The incubation period is short enough to be negligible. Thus the model based on the above assumption is







where is the infection rate and is the removal rate of infectives. This model is called the classic Kermack-Mckendrick model.

Since  be the population size so,



Thus ,  and  are all bounded above by . The initial conditions of the above model are

, , 

From (2), we have



Since from (1) we have

if

then

for all  in which case  as .

So the infection dies out. On the other hand if  then  initially increases and so spread throughout the population.

**Question-03:** Describe the classic-Kermack-Mckendric SIR epidemic model. Discuss the spread of the infection according to this model and how it develops with time

**OR**

Describe the Kermack-Mckendric SIR epidemic model. Define the relative removal rate . Show that if , then  goes monotonically to zero. Discuss .

**Answer:**Let  be the total population size,  be the number of susceptibles,  be the number of infected persons in the population and  the number of those removed from the population by recovery, death or by any other means. The progress of individuals is schematically represented by



Due to infection, the number of susceptibles decreases and the number of infected persons increases. Thus for preparing mathematical model the following assumptions are to be considered.

1. The gain in the infective class is at a rate proportional to the number of infective and susceptibles i.e.  where  is a constant parameter.
2. The rate of removal of infectives to the removed class is proportional to the number of infectives, i.e. , where c is a constant.
3. The incubation period is short enough to be negligible. Thus the model based on the above assumption is







where is the infection rate and is the removal rate of infectives. This model is called the classic Kermack-Mckendrick model.

Since  be the population size so,



Thus ,  and  are all bounded above by . The initial conditions of the above model are

, , 

From (2), we have



Since from (1) we have

if

then

for all  in which case  as .

So the infection dies out i.e. no epidemic can occur. On the other hand if  then  initially increases and we have an epidemic. The term epidemic means that  for some .

Again from (1) and (2) we have





where is relative removal rate.

Integrating,





Using (5) in (6) we get





Putting this value in (6) we have





From (5)  will be maximum if





, since 

Putting  in (7) we get



, since 

If and , then the phase trajectory start with . Also in this case  increases from  and hence an epidemic ensures. If ,  decreases from  and as such epidemic occurs.

**Question-04:** Describe the Kermack-Mckendric SIR epidemic model. Define the relative removal rate . Show that if , then  goes monotonically to zero. Discuss .

**Answer:**Let  be the total population size,  be the number of susceptibles,  be the number of infected persons in the population and  the number of those removed from the population by recovery, death or by any other means. The progress of individuals is schematically represented by



Due to infection, the number of susceptibles decreases and the number of infected persons increases. Thus for preparing mathematical model the following assumptions are to be considered.

1. The gain in the infective class is at a rate proportional to the number of infective and susceptibles i.e.  where  is a constant parameter.
2. The rate of removal of infectives to the removed class is proportional to the number of infectives, i.e. , where c is a constant.
3. The incubation period is short enough to be negligible. Thus the model based on the above assumption is







where is the infection rate and is the removal rate of infectives. This model is called the classic Kermack-Mckendrick model.

Since  be the population size so,



Thus ,  and  are all bounded above by . The initial conditions of the above model are

, , 

From (2), we have



where is relative removal rate.

Since from (1) we have

if

then

for all  in which case  as .

So the infection dies out i.e. no epidemic can occur.

Again from (1) and (3) we have





Integrating,





Using (5) in (6) we get



Putting this value in (6) we have















**Question-05:** Discuss the simple SIS epidemic model.

**Answer:**Let  be the total population size,  be the number of susceptibles and  be the number of infected persons in the population.

Since there is no removal in the population so we have



The initial conditions are



In this model, a susceptible person can become infected at a rate proportional to  and an infected person can recover and become susceptible again at a rate . So the epidemic model is





where is the infection rate and is the removal rate of infectives.

From (2) and (3), we have









Integrating,









By using the initial conditions (3) in (4), we have











Putting this value of  in (4), we get











As ,



**Question-06:** Describe a HIV /AIDS model with treatment as an intervention and find its basic reproduction number.

**Answer:1st Part:**Let  be the total population size and  be a constant immigration rate of susceptible males into .

Let The number of susceptibles.

The number of infectious males.

Natural (Non-AIDS- related) death rate.

The transmission probability.

The number of sextual partners.

 Death rate when population is affected by HIV.

The flow chart is given in below:

Infectious class (Y)

Susceptibles (X)





Natural death





Based on the above diagram the differential equations of a HIV model are



**2nd part: Treatment:**For the HIV treatment, first in 1987 medicine named AZT were recommended. After that there are about 30 medicines recommended for the treatment. Some of them are,

1. The Coctail
2. Antiventrouals
3. Highly active antiventrouals therapy.

Recently five effective medicines are available in the market. Still now, there is no medicine for completely cure from HIV.

**3rd part:**Basic reproductive number is the number of secondary infections which arise from a primary infection.



That is, probability of transmission of the disease



, where  is unit time



.



**Question-01:** Discuss a simple non-linear infection model and explain the dynamics of the model.